① $S = \{ x\langle 2,2,0\rangle + y\langle -1,1,0\rangle \mid x,y \in \mathbb{R} \}$

3 possibilities:

<u>$x=0, y=0$:</u> $0\langle 2,2,0\rangle + 0\langle -1,1,0\rangle = \boxed{\langle 0,0,0\rangle}$

<u>$x=1, y=0$:</u> $1\cdot\langle 2,2,0\rangle + 0\langle -1,1,0\rangle = \boxed{\langle 2,2,0\rangle}$

<u>$x=0, y=1$:</u> $0\cdot\langle 2,2,0\rangle + 1\cdot\langle -1,1,0\rangle = \boxed{\langle -1,1,0\rangle}$

---

②

(a) $-\vec{a} + 2\vec{b} = -\langle 2,1,-2\rangle + 2\langle 1,0,1\rangle$

$= \langle -2,-1,2\rangle + \langle 2,0,2\rangle$

$= \boxed{\langle 0,-1,4\rangle}$

(b) $\|\vec{d}\| = \sqrt{(-1)^2 + (2)^2 + (3)^2 + (-2)^2 + (-1)^2}$

$= \sqrt{1+4+9+4+1} = \boxed{\sqrt{19}}$

(d) $\vec{a}\cdot\vec{b} = \langle 2,1,-2\rangle \cdot \langle 1,0,1\rangle = (2)(1) + (1)(0) + (-2)(1)$

$= \boxed{0}$

$\vec{c}\cdot\vec{d} = \langle 1,1,1,1,1\rangle \cdot \langle -1,2,3,2,-1\rangle$

$= -1+2+3+2-1 = \boxed{5}$

③ (a)

$$2A+B = 2\begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 4 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 5 \end{pmatrix} = \boxed{\begin{pmatrix} 2 & 0 \\ 5 & 9 \end{pmatrix}}$$

(b)

$$AB = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 0-1 & 2-5 \\ 0+2 & 4+10 \end{pmatrix} = \boxed{\begin{pmatrix} -1 & -3 \\ 2 & 14 \end{pmatrix}}$$

(c)

$$CD = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1+0+1 \\ 2+0-3 \end{pmatrix} = \boxed{\begin{pmatrix} 2 \\ -1 \end{pmatrix}}$$

(d) $$E^T = \boxed{\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}}$$

$$C^T = \boxed{\begin{pmatrix} 1 & 2 \\ 1 & 2 \\ -1 & 3 \end{pmatrix}}$$

4

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -3 & -3 & -3 \\ 3 & 2 & 2 & 1 \end{array}\right) \xrightarrow[-3R_1+R_3\to R_3]{-2R_1+R_2\to R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -5 & -5 & -5 \\ 0 & -1 & -1 & -2 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{5}R_2\to R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & -2 \end{array}\right)$$

$$\xrightarrow{R_2+R_3\to R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

$$\boxed{\begin{aligned} x + y + z &= 1 \\ y + z &= 1 \\ 0 &= -1 \end{aligned}}$$

Answer:

no solution

(5) $$\boxed{\begin{aligned} x - 2y \quad\quad -3w &= 1 \\ z + 2w &= 0 \\ w &= 1 \end{aligned}}$$

Already reduced.

leading variables: x, z, w

free variables: y

$$\boxed{\begin{aligned} x &= 1 + 2y + 3w && (1) \\ z &= -2w && (2) \\ w &= 1 && (3) \\ y &= t && (4) \end{aligned}}$$

$$\boxed{\begin{aligned} &(4)\quad y = t \\ &(3)\quad w = 1 \\ &(2)\quad z = -2(1) = -2 \\ &(1)\quad x = 1 + 2t + 3(1) = 4 + 2t \end{aligned}}$$

Answer:

$$\boxed{\begin{aligned} x &= 4 + 2t \\ y &= t \\ w &= 1 \\ z &= -2 \end{aligned}}$$

where t can be any real #

6 See HW 1 - Part 2
Problem 2(e)